# II B. Tech I Semester Supplementary Examinations, May/June - 2016 COMPLEX VARIABLES AND STATISTICAL METHODS 

(Electrical and Electronics Engineering)
Time: 3 hours
Note: 1. Question Paper consists of two parts (Part-A and Part-B)
2. Answer ALL the question in Part-A
3. Answer any THREE Questions from Part-B

## PART -A

1. a) Show that the complex variable function $f(z)=|z|^{2}$ is differentiable only at the origin.
b) Expand the following function in a Taylor's series $\frac{z-1}{z+1}$ about $\mathrm{z}=0$.
c) Determine the poles and residues at each pole of the function $f(z)=\cot z$
d) Find the image of following region in the z-plane onto the w-plane under the given mapping $|z|<1$, Im $z>0 ; w=z+(2+i)$
e) For the discrete probability distribution, find the value of K

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $f(x)$ | 0 | $K$ | 2 K | 2 K | 3 K | $\mathrm{~K}^{2}$ | $2 \mathrm{~K}^{2}$ | $7 \mathrm{~K}^{2}+\mathrm{K}$ |

f) An automobile manufacturer asserts that the seat belts of his seats are $90 \%$ effective. A consumer group tests the seat belts on 50 cars and finds it effective on 37 of them. What is the test statistic to be used to test his hypothesis?

## PART -B

2. a) (i) Show that the function $f(z)=\bar{z}$ is continuous everywhere but not differentiable at any point in the complex plane.
(ii) Show that $f(z)=\bar{z}$ is not differentiable at $z=0$ and is nowhere analytic.
b) Find the analytic function $f(z)=u+i v$ given
$u+v=\frac{2 \sin 2 x}{e^{2 y}+e^{-2 y}-2 \cos 2 x}$
3. a) State and prove Cauchy's integral formula and hence find the value of
(i) $F(3.5)$ (ii) $F(i)$, if
$F(a)=\int_{c} \frac{4 z^{2}+z+5}{z-a} d z$ where c is the ellipse $\left(\frac{x}{2}\right)^{2}+\left(\frac{y}{3}\right)^{2}=1$.
b) Expand $f(z)=\frac{1}{z\left(z^{2}-3 z+2\right)}$ in the region $0<|z|<1$
4. a) State residue theorem and use it to evaluate
$\int_{c} \frac{d z}{z^{8}(z+4)} \quad$ where $c$ is the circle (i) $|z|=2$ (ii) $|z+2|=3$
b) Evaluate $\int_{0}^{2 \pi} \frac{d \theta}{a+b \cos \theta} ; a>|b|>0$ and using it prove that $\int_{0}^{\pi} \frac{d \theta}{17-8 \cos \theta}=\frac{\pi}{15}$.
5. a) Find the image of the triangle with vertices at $i, 1+i, 1-i$ in the z -plane under the transformation
$w=e^{\frac{5 \pi i}{3}} \cdot z-2+4 i$
b) Find the bilinear transformation which maps the points $z=0,1, i$ in the $z$-plane onto the points $1+i,-i, 2-i$ in the w-plane respectively.
6. a) Fit a normal distribution to the following data:

| Class | $60-62$ | $63-65$ | $66-68$ | $69-71$ | $72-74$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 5 | 18 | 42 | 27 | 8 |

b) A random sample of 100 mill workers at Kanpur showed their mean wage to be Rs. 3500 with a standard deviation of Rs. 280. Another random sample of 150 mill workers in Mumbai showed the mean wage to be Rs. 3900 with a standard deviation of Rs. 400. Do the mean wage of the workers in Mumbai and Kanpur differ significantly, at $5 \%$ level of significance?
7. a) Certain pesticides is packed into bags by a machine. A random sample of 10 bags is drawn and their contents are found to weigh in kgs as follows: $50,49,44,52,45,48,46,45,49,45$.
Test if the average picking can be taken as 50 kg .
b) The following data presents the yields in quintals of common 10 subdivisions of equal area of two agricultural plots:

| Plot1 | 6.2 | 5.7 | 6.5 | 6.0 | 6.3 | 5.8 | 5.7 | 6.0 | 6.0 | 5.8 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Plot 2 | 5.6 | 5.9 | 5.6 | 5.7 | 5.8 | 5.7 | 6.0 | 5.5 | 5.7 | 5.5 |

Test whether the two samples taken from the two random populations have the same variance.

